STEP 1: A coin is flipped with probability of heads equal to α.

STEP 2: If the coin comes up heads, the customer enters lottery P_1; otherwise, the customer enters lottery P_2.

Denote this compound lottery by $\alpha P_1 + (1-\alpha)P_2$. Note this compound lottery is also contained in the set $\mathcal{P}$ (i.e., $\mathcal{P}$ is a convex set). We then require the following consistency properties on a customers preference for lotteries:

- **Substitution axiom** For all P_1, P_2, and P_3 in $\mathcal{P}$ and all $\alpha \in (0,1]$, if $P_1 \succ P_2$, then $\alpha P_1 + (1-\alpha)P_3 \succ \alpha P_2 + (1-\alpha)P_3$.
- **Continuity axiom** For all P_1, P_2, and P_3 in $\mathcal{P}$ with $P_1 \succ P_2 \succ P_3$, there exist values $\alpha \in (0,1)$ and $\beta \in (0,1)$ such that $\alpha P_1 + (1-\alpha)P_3 \succ P_2 \succ \beta P_1 + (1-\beta)P_3$.

Roughly, the first axiom says that if one gamble produces strictly preferred outcomes for any realization of uncertainty, then the customer should strictly prefer it. The second axiom says that if a customer strictly prefers one gamble to another, then he should be willing to accept a sufficiently small risk of an even worse outcome to take the preferred gamble. Both are reasonable assumptions.

Under these two axioms, there exist utilities on outcomes such that the expected utility of each lottery defines a customer's preference relation among lotteries. Specifically,

THEOREM E.4 *A preference relation on the lotteries $\mathcal{P}$ exists that satisfies the substitution and continuity axioms if and only if there exists a utility function $u(\cdot)$ such that $P_1 \succ P_2$ if and only if*

$$\sum_{i=1}^{n} u(x_i)P_1(x_i) > \sum_{i=1}^{n} u(x_i)P_2(x_i).$$

That is, if and only if the expected utility from lottery P_1 exceeds the expected utility of lottery P_2. In addition, any two utility functions u and u' satisfying the above must be affine transformations of each other; that is,

$$u(x) = cu'(x) + d,$$

for some real $c > 0$ and d.

This result is due to von Neumann and Morgenstern [541] and is known as the *von Neumann-Morgenstern expected-utility theory.* Essentially, it allows us to extend utility as a model of customer preference to the case of uncertain outcomes, with expected utility replacing deterministic utility as the criterion for customer decision making. Since the original deterministic outcomes (e.g., outcome x_i occurs with probability $P(x_i) = 1$) are included in $\mathcal{P}$, the von Neumann-Morgenstern expected utilities also help us "narrow down" the list of possible utility functions for the customer.

Risk Preferences

An important special case of expected-utility theory is when outcomes represent different monetary amounts, so alternatives correspond to different levels of wealth and lotteries correspond to different gambles on a customer's ending wealth level. For